

Two dimensional fractional supersymmetric conformal field theories and the two point functions

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Abstract

A general two dimensional fractional supersymmetric conformal field theory is investigated. The structure of the symmetries of the theory is studied. Then, applying the generators of the closed subalgebra generated by $(L_{-1}, L_0, G_{-1/3})$ and $(\bar{L}_{-1}, \bar{L}_0, \bar{G}_{-1/3})$, the two point functions of the component fields of supermultiplets are calculated.

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1 Introduction

2D conformally-invariant field theories have become the subject of intense investigation in recent years, after the work of Belavin, Polyakov, and Zamolodchikov [1]. One of the main reasons for this, is that 2D conformal field theories describe the critical behaviour of two dimensional statistical models [2–5]. Conformal field theory provides us with a simple and powerful means of calculating the critical exponents, as well as, the correlation functions of the theory at the critical point [1,6]. Another application of conformal field theories is in string theories. Originally, string theory was formulated in flat twenty six dimensional space-time for bosonic- and flat ten dimensional space-time for supersymmetric-theories. It has been realized now that the central part of string theory is a 2D conformally-invariant field theory. It is also seen that tree level string amplitudes may be expressed in terms of correlation functions of the corresponding conformal field theory on the plane, whereas string loop amplitudes may be expressed in terms of correlation functions of the same conformal field theory on higher genus Riemann surfaces [7–10].

Supersymmetry is a Z_2 extension of the Poincaré algebra [11,12]. But this can be enlarged, to a superconformal algebra for example [13]. If the dimension of the space-time is two, there are also fractional supersymmetric extensions of the Poincaré and conformal algebra [14–17]. Fractional supersymmetry is a Z_n extension of the Poincaré algebra.

In this paper, the special case $n = 3$ is considered. So the components of the superfield have grades 0, 1, and 2. The complex plane is extended by introducing two independent paragrassmann variables θ and $\bar{\theta}$, satisfying $\theta^3 = \bar{\theta}^3 = 0$. One can develop an algebra, the fractional $n = 3$ algebra, based on these variables and their derivatives [18–20]. In [21,22], this fractional supersymmetry has been investigated by introducing a certain fractional superconformal action. We don't consider any special action here. What we do, is to use only the structure of fractional superconformal symmetry in order to obtain general restrictions on the two-point functions. The scheme of the paper is the following. In section 2, infinitesimal superconformal transformations are defined. In section 3, the generators of these transformation and their algebra are investigated. In section 4, the two-point functions of such theories are obtained. Finally, section 5 contains the concluding remarks.

2 Infinitesimal superconformal transformations

Consider a paragrassmann variable θ , satisfying

$$\theta^3 = 0. \quad (1)$$

A function of a complex variable z , and this paragrassmann variable, will be of the form

$$f(z, \theta) = f_0(z) + \theta f_1(z) + \theta^2 f_2(z). \quad (2)$$

Now define the covariant derivatives as [16,18,23,24]

$$D := \partial_\theta - q\theta^2 \partial_z, \quad (3)$$

where q is one of the third roots of unity, not equal to one, and ∂_θ satisfies

$$\partial_\theta \theta = 1 + q\theta \partial_\theta. \quad (4)$$

An infinitesimal transformation

$$\begin{aligned} z' &= z + \omega_0(z) + \theta \omega_1(z) + \theta^2 \omega_2(z) \\ \theta' &= \theta + \epsilon_0(z) + \theta \epsilon_1(z) + \theta^2 \epsilon_2(z) \end{aligned} \quad (5)$$

is called superconformal if

$$D = (D\theta')D' \quad (6)$$

where

$$D' = \partial_{\theta'} - q\theta'^2 \partial_{z'} \quad (7)$$

From these, it is found that an infinitesimal superconformal transformation is of the form

$$\begin{aligned} \theta' &= \theta + \epsilon_0(z) + \frac{1}{3}\theta \omega'_0(z) + \theta^2 \epsilon_2(z), \\ z' &= z + \omega_0(z) - q\theta^2 \epsilon_0(z), \end{aligned} \quad (8)$$

where $\omega'_0(z) := \partial_z \omega_0(z)$. It is also seen that the following commutative relations hold.

$$\begin{aligned} \epsilon_2 \theta &= q\theta \epsilon_2, \\ \epsilon_0 \theta &= q\theta \epsilon_0. \end{aligned} \quad (9)$$

One can extend these naturally to functions of z and \bar{z} , and θ and $\bar{\theta}$ (full functions instead of chiral ones). It is sufficient to define a covariant derivative

for the pair $(\bar{z}, \bar{\theta})$, the analogue of (3), and extend the transformations (5), so that there are similar transformations for $(\bar{z}, \bar{\theta})$ as well. Then, defining a superconformal transformation as one satisfying (6) and its analogue for $(\bar{z}, \bar{\theta})$, one obtains, in addition to (8) and (9), similar expressions where $(z, \theta, \omega, \epsilon)$ are simply replaced by $(\bar{z}, \bar{\theta}, \bar{\omega}, \bar{\epsilon})$. So, the superconformal transformations consist of two distinct class of transformations, the holomorphic and the antiholomorphic, that do not talk to each other.

3 Generators of superconformal field theory

The (chiral) superfield $\phi(\theta, z)$ with the expansion [24]

$$\phi(\theta, z) = \varphi_0(z) + \theta\varphi_1(z) + \theta^2\varphi_2(z) \quad (10)$$

is a super-primaryfield of weight Δ if it transforms under a superconformal transformations as

$$\phi(\theta, z) \mapsto (D\theta')^{3\Delta} \phi(\theta', z') \quad (11)$$

One can write this as

$$\phi(\theta, z) \mapsto [1 + \tilde{T}(\omega_0) + \tilde{S}(\epsilon_0) + \tilde{H}(\epsilon_2)]\phi(\theta, z), \quad (12)$$

to arrive at [18]

$$\begin{aligned} \tilde{T}(\omega_0) &= \omega_0 \partial_z + \left(\Delta + \frac{\Lambda}{3} \right) \omega'_0, \\ \tilde{S}(\epsilon_0) &= \epsilon_0 (\delta_\theta - q^2 \theta^2 \partial_z) - 3\Delta q^2 \epsilon'_0 \theta^2, \\ \tilde{H}(\epsilon_2) &= q\epsilon_2 (\theta^2 \partial_\theta - \Delta \theta). \end{aligned} \quad (13)$$

Here Λ and δ_θ are operators satisfying

$$[\Lambda, \theta] = 1, \quad (14)$$

and

$$\delta_\theta \theta = q^{-1} \theta \delta_\theta + 1, \quad (15)$$

respectively.

One can now define the generators

$$\begin{aligned} l_n &:= \tilde{T}(z^{n+1}), \\ g_r &:= \tilde{S}(z^{r+1/3}), \end{aligned} \quad (16)$$

where n and $r + 1/3$ are integers. The generators of superconformal transformations are defined through

$$\begin{aligned} [L_n, \phi(\theta, z)] &:= l_n \phi, \\ [G_r, \phi(\theta, z)] &:= g_r \phi. \end{aligned} \quad (17)$$

One can check that, apart from a possible central extension, these generators satisfy the following relations.

$$[L_n, L_m] = (n - m) L_{n+m}, \quad (18)$$

$$[L_n, G_r] = \left(\frac{n}{3} - r\right) G_{n+r}, \quad (19)$$

and

$$G_r G_s G_t + \text{five other permutations of the indices} = 6 L_{r+s+t}. \quad (20)$$

This algebra, which contains the Virasoro algebra (18) as a subalgebra, has nontrivial central extensions. It is shown [16,25] that there is only one subalgebra (containing G -generators as well as L_n 's), the central extension for which is trivial. This algebra is the one generated by $\{L_{-1}, L_0, G_{-1/3}\}$. Note that if one excludes G_r 's, there exists another subalgebra generated by $\{L_{-1}, L_0, L_1\}$, the central extension of which is trivial. Also note that we have not included the generators corresponding to \tilde{H} in the algebra. The reason is that there is no closed subalgebra, with a trivial central extension, containing these generators [16,25].

Now we have the effects of L_n 's and G_r 's on the superfield. It is not difficult to obtain their effect on the component fields. For L_n 's, the first equation of (17) leads directly to

$$[L_n, \phi_k(z)] = z^{n+1} \partial_z \phi_k + (n+1) z^n \left(\Delta + \frac{k}{3} \right) \phi_k. \quad (21)$$

This shows that the component field ϕ_k is simply a primary field with the weight $\Delta + k/3$. One can also write (21) in terms of operator-product expansion:

$$\mathcal{R}[T(w)\phi_k(z)] \sim \frac{\partial \phi_k(z)}{w-z} + \frac{(\Delta + k/3)\phi_k(z)}{(w-z)^2}, \quad (22)$$

where \mathcal{R} denotes the radial ordering and $T(z)$ is the holomorphic part of the energy-momentum tensor:

$$T(z) = \sum_n \frac{L_n}{z^{n+2}}. \quad (23)$$

For G_r 's, a little more care is needed. One defines a χ -commutator as [26]

$$[A, B]_\chi := AB - \chi BA. \quad (24)$$

It is easy to see that

$$[A, BB']_{\chi\chi'} = [A, B]_\chi B' + \chi B[A, B']_{\chi'}. \quad (25)$$

Now, if we use

$$[G, \theta]_q = 0, \quad (26)$$

then the second equation of (17) leads to

$$\begin{aligned} [G_r, \phi_0(z)] &= z^{r+1/3} \phi_1, \\ [G_r, \phi_1(z)]_{q^{-1}} &= -z^{r+1/3} \phi_2, \\ [G_r, \phi_2(z)]_{q^{-2}} &= -\left[z^{r+1/3} \partial_z \phi_0 + \left(r + \frac{1}{3} \right) z^{r-2/3} (3\Delta) \phi_0 \right]. \end{aligned} \quad (27)$$

This can also be written in terms of the operator-product expansion. To do this, however, one should first define a proper radial ordering for the supersymmetry generator and the component fields. Defining

$$\mathcal{R}[S(w)\phi_k(z)] := \begin{cases} S(w)\phi_k(z), & |w| > |z| \\ q^{-k}\phi_k(z)S(w), & |w| < |z| \end{cases} \quad (28)$$

where

$$S(z) := \sum_r \frac{G_r}{z^{r+4/3}}, \quad (29)$$

one is led to

$$\begin{aligned} \mathcal{R}[S(w)\phi_0(z)] &\sim \frac{\phi_1(z)}{w-z}, \\ \mathcal{R}[S(w)\phi_1(z)] &\sim -\frac{\phi_2(z)}{w-z}, \\ \mathcal{R}[S(w)\phi_2(z)] &\sim -\frac{\partial\phi_0(z)}{w-z} - \frac{3\Delta\phi_0(z)}{(w-z)^2}. \end{aligned} \quad (30)$$

What we really use to restrict the correlation functions is that part of the algebra the central extension of which is trivial, that is, the algebra generated by $\{L_{-1}, L_0, G_{-1/3}\}$.

4 Two-point functions

The two-point functions should be invariant under the action of the subalgebra generated by $\{L_{-1}, L_0, G_{-1/3}\}$. This means

$$\langle 0|[L_{-1}, \phi_k \phi'_{k'}]|0\rangle = 0, \quad (31)$$

$$\langle 0|[L_0, \phi_k \phi'_{k'}]|0\rangle = 0, \quad (32)$$

$$\langle 0|[G_{-1/3}, \phi_k \phi'_{k'}]_{q^{-k-k'}}|0\rangle = 0. \quad (33)$$

Here we have used the shorthand notation $\phi_k = \phi_k(z)$ and $\phi'_{k'} = \phi'_{k'}(z')$. ϕ and ϕ' are primary superfields of weight Δ and Δ' , respectively. Equations (31) and (32) imply that

$$\langle \phi_k \phi'_{k'} \rangle = \frac{A_{k,k'}}{(z - z')^{\Delta + \Delta' + (k+k')/3}}. \quad (34)$$

This is simply due to the fact that ϕ_k and $\phi'_{k'}$ are primary fields of the weight $\Delta + k/3$ and $\Delta' + k'/3$, respectively. Note that it is not required that these weights be equal to each other, since we have not included L_1 in the subalgebra.

(33) relates A_{k_1, k'_1} with A_{k_2, k'_2} , if

$$k_1 + k'_1 - (k_2 + k'_2) = 0, \quad \text{mod } 3. \quad (35)$$

Therefore, there remains 3 independent constants in the 9 correlation functions. Correlation functions of grade 0:

$$\begin{aligned} \langle \phi_0 \phi'_0 \rangle &:= A_0 f_{0,0} = A_0 \frac{1}{(z - z')^{\Delta + \Delta'}}, \\ \langle \phi_1 \phi'_2 \rangle &:= A_0 f_{1,2} = A_0 \frac{\Delta + \Delta'}{(z - z')^{\Delta + \Delta' + 1}}, \\ \langle \phi_2 \phi'_1 \rangle &:= A_0 f_{2,1} = A_0 \frac{-q^2(\Delta + \Delta')}{(z - z')^{\Delta + \Delta' + 1}}, \end{aligned} \quad (36)$$

those of grade one:

$$\begin{aligned} \langle \phi_0 \phi'_1 \rangle &:= A_1 f_{0,1} = A_1 \frac{1}{(z - z')^{\Delta + \Delta' + 1/3}}, \\ \langle \phi_1 \phi'_0 \rangle &:= A_1 f_{1,0} = A_1 \frac{-1}{(z - z')^{\Delta + \Delta' + 1/3}}, \\ \langle \phi_2 \phi'_2 \rangle &:= A_1 f_{2,2} = A_1 \frac{q^2(\Delta + \Delta' + 1/3)}{(z - z')^{\Delta + \Delta' + 4/3}}, \end{aligned} \quad (37)$$

and those of grade two:

$$\langle \phi_0 \phi'_2 \rangle := A_2 f_{0,2} = A_2 \frac{1}{(z - z')^{\Delta + \Delta' + 2/3}},$$

$$\begin{aligned}
\langle \phi_1 \phi'_1 \rangle &:= A_2 f_{1,1} = A_2 \frac{1}{(z - z')^{\Delta + \Delta' + 2/3}}, \\
\langle \phi_2 \phi'_0 \rangle &:= A_2 f_{2,0} = A_2 \frac{q^2}{(z - z')^{\Delta + \Delta' + 2/3}}.
\end{aligned} \tag{38}$$

It is easy to check that adding L_1 to the subalgebra trivializes the correlation functions. The reason is that in all of the correlation sets (of the same grade) there exists at least one correlation function the weights of its primary fields are not equal to each other, regardless of the values of Δ and Δ' . In fact, acting on $\langle \phi_k \phi'_{k'} \rangle$ by $G_{-1/3}$ relates $\langle \phi_{k+1} \phi'_{k'} \rangle$ to $\langle \phi_k \phi'_{k'+1} \rangle$. And it is impossible that both

$$\Delta + 1/3 = \Delta', \quad \text{mod } 1 \tag{39}$$

and

$$\Delta = \Delta' + 1/3, \quad \text{mod } 1 \tag{40}$$

hold. This shows that either $\langle \phi_{k+1} \phi'_{k'} \rangle$ or $\langle \phi_k \phi'_{k'+1} \rangle$ are zero, and so the other should be zero as well.

So far, everything has been calculated for the chiral fields. Let us generalize this to the full fields. The generalization is not difficult. One introduces component fields $\phi_{k\bar{k}}(z, \bar{z})$ which have the following properties. The action of the energy-momentum generators on these component fields is

$$\begin{aligned}
[L_n, \phi_{k\bar{k}}(z, \bar{z})] &= z^{n+1} \partial_z \phi_{k\bar{k}} + (n+1) z^n \left(\Delta + \frac{k}{3} \right) \phi_{k\bar{k}}, \\
[\bar{L}_n, \phi_{k\bar{k}}(z, \bar{z})] &= \bar{z}^{n+1} \partial_{\bar{z}} \phi_{k\bar{k}} + (n+1) \bar{z}^n \left(\bar{\Delta} + \frac{\bar{k}}{3} \right) \phi_{k\bar{k}}.
\end{aligned} \tag{41}$$

While there are generators of the supersymmetry which act like

$$\begin{aligned}
[G_r, \phi_{0\bar{k}}(z, \bar{z})]_{q^{-\bar{k}}} &= z^{r+1/3} \phi_{1\bar{k}}, \\
[G_r, \phi_{1\bar{k}}(z, \bar{z})]_{q^{-1-\bar{k}}} &= -z^{r+1/3} \phi_{2\bar{k}}, \\
[G_r, \phi_{2\bar{k}}(z, \bar{z})]_{q^{-2-\bar{k}}} &= - \left[z^{r+1/3} \partial_z \phi_{0\bar{k}} + \left(r + \frac{1}{3} \right) z^{r-2/3} (3\Delta) \phi_{0\bar{k}} \right],
\end{aligned} \tag{42}$$

and

$$\begin{aligned}
[\bar{G}_r, \phi_{k0}(z, \bar{z})]_{q^{-k}} &= \bar{z}^{r+1/3} \phi_{k1}, \\
[\bar{G}_r, \phi_{k1}(z, \bar{z})]_{q^{-k-1}} &= -\bar{z}^{r+1/3} \phi_{k2}, \\
[\bar{G}_r, \phi_{k2}(z, \bar{z})]_{q^{-k-2}} &= - \left[\bar{z}^{r+1/3} \partial_{\bar{z}} \phi_{k0} + \left(r + \frac{1}{3} \right) \bar{z}^{r-2/3} (3\bar{\Delta}) \phi_{k0} \right].
\end{aligned} \tag{43}$$

The equations to be satisfied by the two-point functions are

$$\begin{aligned}\langle 0|[L_{-1}(\bar{L}_{-1}), \phi_{k\bar{k}}\phi'_{k'\bar{k}'}]|0\rangle &= 0, \\ \langle 0|[L_0(\bar{L}_0), \phi_{k\bar{k}}\phi'_{k'\bar{k}'}]|0\rangle &= 0, \\ \langle 0|[G_{-1/3}(\bar{G}_{-1/3}), \phi_{k\bar{k}}\phi'_{k'\bar{k}'}]_{q^{-k-\bar{k}-k'-\bar{k}'}}|0\rangle &= 0.\end{aligned}\tag{44}$$

Once again, the first two sets of equations simply imply that

$$\langle \phi_{k\bar{k}}\phi'_{k'\bar{k}'} \rangle = \frac{A_{k\bar{k}, k'\bar{k}'}}{(z - z')^{\Delta + \Delta' + (k + k')/3} (\bar{z} - \bar{z}')^{\bar{\Delta} + \bar{\Delta}' + (\bar{k} + \bar{k}')/3}}.\tag{45}$$

The third set of equations relate $A_{k_1\bar{k}_1, k'_1\bar{k}'_1}$ to $A_{k_2\bar{k}_2, k'_2\bar{k}'_2}$, provided

$$k_1 + k'_1 - (k_2 + k'_2) = \bar{k}_1 + \bar{k}'_1 - (\bar{k}_2 + \bar{k}'_2) = 0, \quad \text{mod } 3.\tag{46}$$

So, there remains nine arbitrary constants in these correlation functions. One can write the correlations in terms of the correlations of the chiral fields. To see this, notice that

$$\langle 0|[G_{-1/3}, \phi_{k\bar{k}}\phi'_{k'\bar{k}'}]_{q^{-k-\bar{k}}} |0\rangle = O_k \langle \phi_{(k+1)\bar{k}}\phi'_{k'\bar{k}'} \rangle + q^{-k-\bar{k}} O'_{k'} \langle \phi_{k\bar{k}}\phi'_{(k'+1)\bar{k}'} \rangle.\tag{47}$$

Here O_k and $O'_{k'}$ are coefficients (or derivatives) which depend only on k and k' , respectively. The right-hand should of course be zero, due to the supersymmetry of the theory. The corresponding equation for chiral correlations differs from this, only in the coefficient $q^{-k-\bar{k}}$. For the chiral correlations, this coefficient is simply q^{-k} . But it is easy to check that the set $q^{k\bar{k}} \langle \phi_{k\bar{k}}\phi'_{k'\bar{k}'} \rangle$ satisfies the same equations of the chiral correlations. It is also easy to see that this same set satisfy the same equations of the antichiral correlations as well. So, the general solution for the full correlations is

$$\langle \phi_{k\bar{k}}\phi'_{k'\bar{k}'} \rangle = A_{K\bar{K}} q^{-k\bar{k}} f_{k, k'}(z - z') \bar{f}_{\bar{k}, \bar{k}'}(\bar{z} - \bar{z}'),\tag{48}$$

where $f_{k, k'}$'s are defined through (36)–(38), $\bar{f}_{\bar{k}, \bar{k}'}$'s are the same as these with $\Delta \rightarrow \bar{\Delta}$ and $\Delta' \rightarrow \bar{\Delta}'$,

$$K = k + k', \quad \text{mod } 3,\tag{49}$$

and

$$\bar{K} = \bar{k} + \bar{k}', \quad \text{mod } 3.\tag{50}$$

5 Concluding remarks

We considered fractional superconformal field theories only through the general symmetries of such theories. For the special case of three component fields in the superfield, the two-point functions were obtained. In fractional superconformal field theories, one cannot impose the symmetries generated by L_1 and \bar{L}_1 as non anomalous symmetries. Otherwise, the whole theory becomes trivial. This is due to the fact that the fractional supersymmetry generator changes the weight of the fields by $1/3$ (or $1/f$ if there are f component fields in the superfield). So, two two-point functions will be related to each other, and the difference between the weights of the fields of one correlation is $2/f$ greater than the difference between the weights of the fields of the other correlation. As the symmetry generated by L_1 requires the weights of the fields in nonzero two-point functions be equal to each other, at least one of these correlations will be zero, and this forces the other to be zero as well. One can also see that the theory becomes trivial, through the fact that adding L_1 to the subalgebra generated by $\{L_{-1}, L_0, G_{-1/3}\}$ brings all of the generators of the superconformal theory to the algebra.

As L_1 is not in the non anomalous symmetry generators, the component fields don't enjoy the full conformal symmetry (that is the $sl(2)$ symmetry generated by $\{L_{-1}, L_0, L_1\}$). So, the three-point functions cannot be determined up to only constants; there remains unknown functions in them. In fact one has lost the special conformal transformations as non anomalous symmetries, and the symmetries of the theory, apart from the supersymmetry, are just the space-time symmetries plus dilation.

References

- [1] A. A. Belavin, A. M. Polyakov, A. B. Zamolodchikov, Nucl. Phys. B241 (1984) 333
- [2] Manuel Asorey, Fortschr. Phys. 40 (1992) 3, 273
- [3] P. Christe and M. Henkel, Introduction to Conformal Invariance and it's Applications to Critical Phenomena, Springer Lecture Notes in Physics, Vol. m 16 (1993)
- [4] John L. Cardy, Les Houches, session XLIX, 1988 /fields, strings and critical phenomena, ed. E. Brezin and J. Zinn-Justin (North-Holland, Amsterdam, 1990)
- [5] Philippe Di Francesco, Pierre Mathieu and David Senecal, Conformal Field Theory, Graduate Texts in Contemporary Physics, 1997 Springer-Verlag, New York, Inc.
- [6] Daniel Friedan, Zongan Qiu, and Stephen Shenker, Phys. Rev. Lett. 52, (1984) 1575
- [7] P. Ginsparg, Les Houches, session XLIX, 1988 /fields, strings and critical phenomena, ed. E. Brezin and J. Zinn-Justin (North-Holland, Amsterdam, 1990)
- [8] Lance J. Dixon, Introduction to conformal field theory and string theory, slac-pub-5149, 1989
- [9] Elias Kiritsis, Introduction to superstring theory, preprint CERN-TH/97-218, hep-th/9709062
- [10] Joseph Polchinski, What is string theory, preprint NSF-ITP-94-97, hep-th/9411028
- [11] S. Coleman and J. Mandula, Phys. Rev 159 (1967) 1251
- [12] R. Haag, J. T. Lopuszanski and M. F. Sohnius, Nucl. Phys. B88 (1975) 257
- [13] D. Friedan, Z. Qiu and S. Shenker, Phys. Lett. 151B (1985) 37
- [14] C. Ahn, D. Bernard and A. Leclair, Nucl. Phys. B346 (1990) 409

- [15] J. A. de Azcarraga and A. J. Macfarlane, J. Math. Phys. 37 (1996) 111
- [16] S. Durand, Phys. Lett B 312 (1993) 115
- [17] N. Fleury and M. Raush de Trautenberg, Mod. Phys. Lett, A11 (1996) 899
- [18] A. T. Philippov, A. P. Isaev and A. B. Kurdikov, Paragrassmann extensions of the Virasoro algebra, preprint hep-th/9212157
- [19] S. Durand, Mod. Phys. Lett A7 (1992) 2905
- [20] A. T. Filippov and A. B. Kurdikov, Paragrassmann algebras with many variables, preprint hep-th/9312081, A. P. Isaev, Paragrassmann integral, discrete systems and quantum groups, preprint q-alg/9609030
- [21] A. Perez, M. Rausch de Trautenberg, and P. Simon, Nucl. Phys. B 482 (1996) 325
- [22] M. Rausch de Trautenberg and P. Simon, 2D-Fractional supersymmetry and conformal field theory for alternative statistics, preprint hep-th/9606188
- [23] S. Durand, Mod. Phys. Lett A8(1993) 2323
- [24] E H Saidi, M B sedra and Zarouaoui, Class. Quantum. Grav. 12 (1995) 1567
- [25] S. Durand and L. Vinet, Mod. Phys. Lett A4, 2519 (1989)
- [26] R. S. Dunne, A. J. Macfarlane, J. A. Azcarraga et J. C. Perez Bueno, Int. J. Mod. Phys. A12 (1997) 3275
- [27] Zongan Qiu, Nucl. Phys. B 270 [Fs16] (1986) 205